Simplifying and Solving

Upper Moreland High School Math Department

CHAPTER 3 Simplifying and Solving

In this chapter you will focus on multiplying expressions. You will also solve equations that contain products. While these new ideas will be introduced using algebra tiles, you will also develop a method to multiply expressions without using tiles.

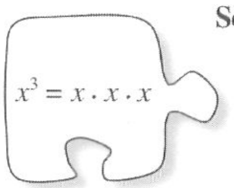
Guiding Question

Mathematically proficient students use appropriate tools strategically.

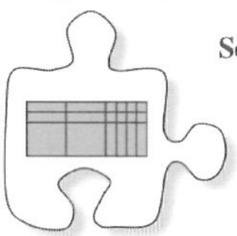
As you work through this chapter, ask yourself:

How can algebra tiles and area models help me better understand multiplication?

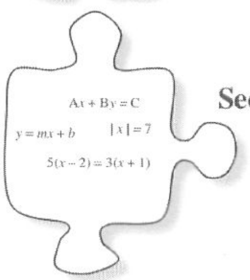
Chapter Outline



Section 3.1 You will simplify expressions with exponents by using the number 1.



Section 3.2 You will learn how to use algebra tiles to physically and visually represent an equation. You will also make another equation ↔ situation connection on the multiple representations web. Then, using algebra tiles and generic rectangles, you will develop a method to rewrite products of binomials and other polynomials, such as $(3x-2)(4+x)$.



Section 3.3 You will solve one-variable equations containing products and absolute value, and you will learn how to solve multi-variable equations for one of the variables.

3.1.1 How can I rewrite it?

Simplifying Exponential Expressions

$x^3 = x \cdot x \cdot x$

Today you will examine how to simplify expressions with exponents. Using patterns, you will develop strategies to simplify expressions when the exponents are too large to expand on paper.

3-1. An **exponent** is shorthand for repeated multiplication. For example, $n^4 = n \cdot n \cdot n \cdot n$. In an exponential expression like b^a, b is called the **base** and a is called the **exponent**.

Expand each of the expressions below. For example, to expand x^3, you would write: $x \cdot x \cdot x$.

a. y^7 b. $5(2m)^3$ c. $(x^3)^2$ d. $4x^5y^2$

3-2. Ms. Wang has just explained to her class how to simplify exponents by using the number 1. She wrote the following on the board:

$$\begin{aligned} &\frac{xy^8}{y^2} \\ &= \frac{x}{1} \cdot \frac{y^2}{y^2} \cdot \frac{y^6}{1} \\ &= \frac{x}{1} \cdot 1 \cdot \frac{y^6}{1} \\ &= \frac{xy^6}{1} \\ &= xy^6 \end{aligned}$$

a. Copy Ms. Wang's steps on to your paper. Explain each step.

b. Simplify each of the expressions below using what you know about exponents and the number 1. Start by expanding the exponents, and then simplify your results.

i. $\frac{x \cdot x \cdot x}{x}$ *ii.* $\frac{x^5}{x^2}$ *iii.* $x^2 \cdot x^3$ *iv.* $k^3 \cdot k^5$

v. $\frac{16k^3}{8k^2}$ *vi.* $m^6 \cdot m$ *vii.* $x^4 \cdot x^5 \cdot x^3$ *viii.* $\frac{6x^3y}{2y}$

challenge: $\frac{5x^{50}}{10x^{15}}$

3-3. Simplify each of the expressions below. Start by expanding the exponents, and then simplify your results. Look for patterns or possible shortcuts that will help you simplify more quickly. Be prepared to justify your patterns or shortcuts to the class.

a. $y^5 \cdot y^2$

b. $\frac{w^5}{w^2}$

c. $(x^2)^4$

d. $x^{10} \cdot x^{12}$

e. $\frac{13p^4q^5}{p^2q^2}$

f. $\left(\frac{x^2}{y}\right)^3$

g. $5h \cdot 2h^{24}$

h. $\frac{10m^{30}}{2m^8}$

i. $(3k^{20})^4$

j. $\frac{24hg^2}{3hg^9}$

k. $\left(\frac{m^3}{n^{10}}\right)^4$

l. $w^4 \cdot p \cdot w^3$

3-4. Work with your team to write four exponent problems, each having a simplification of x^{12}. At least one problem must involve multiplication, one must involve grouping, and one must involve division. Be creative!

3-5. Lacey and Haley are simplifying expressions.

a. Haley simplified $x^3 \cdot x^2$ and got x^5. Lacey simplified $x^3 + x^2$ and got the same result! However, their teacher told them that only one simplification is correct. Who simplified correctly and how do you know?

b. Haley simplifies $3^5 \cdot 4^5$ and gets the result 12^{10}, but Lacey is not sure. Is Haley correct? Be sure to justify your answer.

Review & Preview

3-6. Use what you have learned about exponents to rewrite each of the expressions below.

a. $\frac{h^9}{h^{11}}$ b. $x^3 \cdot x^4$ c. $(3k^5)^2$

d. $n^7 \cdot n$ e. $\frac{16x^4y^3}{2x^4}$ f. $4xy^3 \cdot 7x^2y^3$

3-7. Gerardo is simplifying expressions with very large exponents. He arrives at each of the results below. For each result, decide if he is correct and justify your answer using the meaning of exponents.

a. $\frac{x^{150}}{x^{50}} \Rightarrow x^3$

b. $y^{20} \cdot y^{41} \Rightarrow y^{61}$

c. $(2m^2n^{15})^3 \Rightarrow 2m^6n^{45}$

3-8. Use what you know about slope and y-intercept to graph $y = -\frac{1}{2}x + 3$.

3-9. Write an expression to represent the given situation. Be sure to define your variable.

Sam currently has \$150 in a savings account and is saving \$10 per week.

3-10. Find $f(-3)$ for each function below.

a. $f(x) = -2x + 3$ b. $f(x) = -|1 - x|$

c. $f(x) = \sqrt[3]{9x} + 2$ d. $f(x) = \frac{1}{2}x + 2$

3-11. Simplify each expression.

a. $-\frac{1}{2} + \left(-\frac{1}{5}\right)$ b. $-\frac{2}{3} - 2$

c. $-1\frac{2}{3}(-2)$ d. $-2 \div \frac{2}{3}$

3.1.2 How can I rewrite it?

$x^3 = x \cdot x \cdot x$

Zero and Negative Exponents

In Lesson 3.1.1, you used the meaning of an exponent to rewrite expressions such as $y^4 \cdot y^2$ and $(x^2y)^3$. Today you will use the patterns you discovered to learn how to interpret expressions with exponents that are negative or zero.

3-12. Review what you learned about exponents in Lesson 3.1.1 to rewrite each expression below as simply as possible. If you see a pattern or know of a shortcut, be sure to share it with your teammates.

a. $x^7 \cdot x^4$ b. $(x^3)^3$ c. $\frac{m^{14}}{m^2}$

d. $(x^2y^2)^4$ e. $\frac{x^2y^{11}}{x^5y^3}$ f. $\frac{2x^{12}}{8x^2}$

3-13. With your study team, summarize the patterns you found in problem 3-12. For each one, simplify the given expression and write an expression that represents its generalization. Then, in your own words, explain why the pattern works.

	Expression	Generalization	Why is this true?
a.	$x^{25} \cdot x^{40} = ?$	$x^m \cdot x^n = ?$	
b.	$\frac{x^{36}}{x^{13}} = ?$	$\frac{x^m}{x^n} = ?$	
c.	$(x^5)^{12} = ?$	$(x^m)^n = ?$	

3-14. Describe everything you know about $\frac{x^m}{x^m}$. What is its value? How can you rewrite it using a single exponent? What new conclusions can you draw? Be prepared to explain your findings to the class.

3-15. Problem 3-14 helped you recognize that $x^0 = 1$. Now you will similarly use division to explore the meaning of x^{-1}, x^{-2}, etc. Simplify each of the expressions below *twice*:

- Once by expanding the terms and simplifying.
- Again by using your new pattern for division with exponents.

Be ready to discuss the meaning of negative exponents with the class.

a. $\frac{x^4}{x^5}$ b. $\frac{x^2}{x^4}$ c. $\frac{x^7}{x^{10}}$

3-16. Use your exponent patterns to rewrite each of the expressions below. For example, if the original expression has a negative exponent, then rewrite the expression so that it has no negative exponents, and vice versa. Also, if the expression contains multiplication or division, then use your exponent rules to simplify the expression.

a. k^{-5} b. m^0 c. $x^{-2} \cdot x^5$ d. $\frac{1}{p^2}$

e. $\frac{y^{-2}}{y^{-3}}$ f. $(x^{-2})^3$ g. $(a^2b)^{-1}$ h. $\frac{1}{x^{-1}}$

3-17. EXPONENT CONCENTRATION

Split your team into two pairs and decide which is Team A and which is Team B. Your teacher will distribute a set of cards for a game described below.

- Arrange the cards face down in a rectangular grid.
- Team A selects and turns over two cards.
- If Team A thinks the values on the cards are equivalent, they must justify this claim to Team B. If everyone in Team B agrees, Team A takes the pair. If the values are not equivalent, Team A returns both cards to their original position (face down). This is the end of the turn for Team A.
- Team B repeats the process.
- Teams alternate until no cards remain face down. The team with the most matches wins.

3-18. In your Learning Log, describe the meaning of zero and negative exponents. That is, explain how to interpret x^0 and x^{-1}. Title this entry "Zero and Negative Exponents" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Laws of Exponents

In the expression x^3, x is the **base** and 3 is the **exponent**.

$$x^3 = x \cdot x \cdot x$$

The patterns that you have been using during this section of the book are called the **laws of exponents**. Here are the basic rules with examples:

Law	Examples	
$x^m x^n = x^{m+n}$ for all x	$x^3x^4 = x^{3+4} = x^7$	$2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$	$\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$	$(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$	$9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \neq 0$	$\frac{1}{x^2} = (\frac{1}{x})^2 = (x^{-1})^2 = x^{-2}$	$3^{-1} = \frac{1}{3}$

Review & Preview

3-19. Which of the expressions below are equivalent to $16x^8$? Make sure you find *all* the correct answers!

a. $(16x^4)^2$

b. $8x^2 \cdot 2x^6$

c. $(2x^2)^4$

d. $(4x^4)^2$

e. $(2x^4)^4$

f. $(\frac{1}{16}x^{-8})^{-1}$

3-20. Rewrite each expression below without negative or zero exponents.

a. 4^{-1} b. 7^{0} c. 5^{-2} d. x^{-2}

3-21. With or without tiles, simplify, and solve each equation below for x. Record your work.

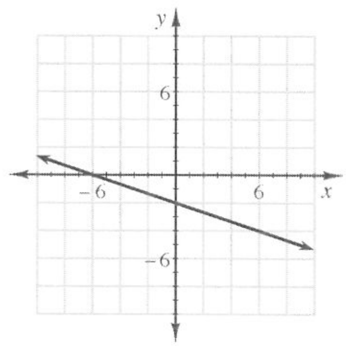
a. $3x-7=2$ b. $1+2x-x=x-5+x$

c. $3-2x=2x-5$ d. $3+2x-(x+1)=3x-6$

3-22. For the line graphed at right:

a. Determine the slope.

b. Find the equation of the line.



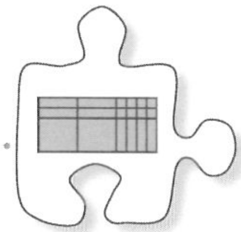
3-23. Write and solve an equation to represent the given situation. Be sure to define your variable.

Samantha currently has \$1500 in the bank and is spending \$35 per week. How many weeks will it take until her account is worth only \$915?

3-24. Determine the equation of the line containing the points given in the table below.

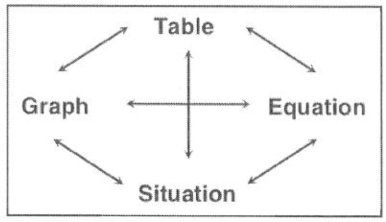
x	−2	−1	2	3
y	−7	−4	5	8

3.2.1 How can I represent an equation?



Equations ↔ Algebra Tiles

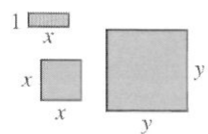
In Chapter 2, you learned about the multiple representations of a linear function, as shown in the web at right. Today you will look more at the equation ↔ situation connection. You will do this by using "algebra tiles" to model them. Algebra tiles are a way to represent an equation physically and visually.



3-25. Your teacher will distribute a set of algebra tiles for your team to use during this course.

a. The tiles have a positive side and a negative side. In this text the positive side will be the shaded side. Flip the tiles so that the positive side of each tile is facing up. Trace one of each of the six tiles provided by your teacher on your paper. Leave plenty of space between each tracing.

b. The dimensions of some of the tiles are shown at right. Label the dimensions of all the tiles next to the tracings you made.

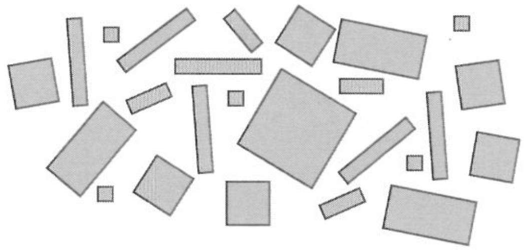


c. The algebra tiles will be named according to each of their areas. Write the name of each tile in the center of your tracing with a colored pen or pencil. Make the name of the tile stand out.

d. Below each tile write "P =" and then find the perimeter of each tile.

3-26. JUMBLED PILES

a. Your teacher will show you a jumbled pile of algebra tiles similar to the one below. Write the shortest description for the collection of tiles on your paper.



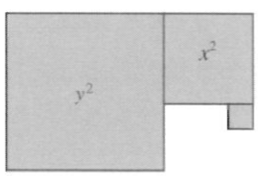
b. Build each collection of tiles below. Then name the collection using the simplest algebraic expression you can.

i. $3x+5+x^2+y+2x^2+2+x$ *ii.* $3y+2+2xy+4x+y^2+4y+1$

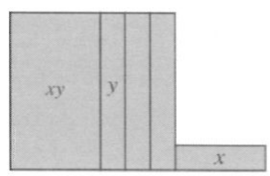
3-27. For each of the shapes formed by algebra tiles below:

- Use tiles to build the shape.
- Sketch and label the shape on your paper.
- Write a simplified expression that represents the perimeter.
- Write a simplified expression that represents the area.

a.

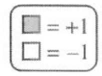


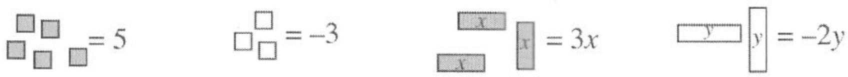
b.



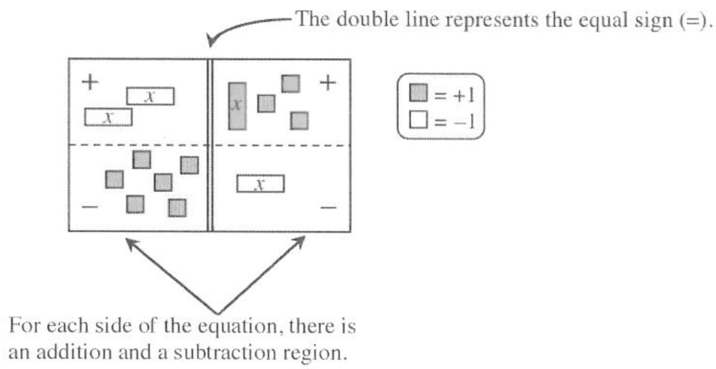
3-28. NEGATIVES AND SUBTRACTION

Let's look at how you can use algebra tiles to represent "negative." Below are several tiles with their associated values. Note that the shaded tiles are positive and the un-shaded tiles are negative (as shown in the diagram at right, which will appear throughout the text as a reminder).

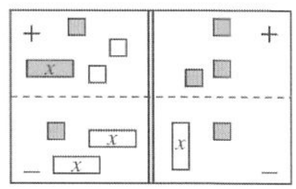




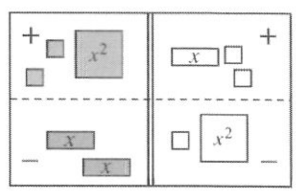
"Subtraction" can be represented with a tool called an **Equation Mat**. For example, the equation $-2x-6=x+3-(-x)$ can be represented by the Equation Mat below.



a. What equation is represented by the Equation Mat at right? Do not simplify the equation; simply write down what you see.



b. What equation is represented by the Equation Mat at right? Do not simplify.



3-29. SOLVING WITH AN EQUATION MAT

a. Obtain the Lesson 3.2.1B Resource Page ("Equation Mat") from your teacher. Build the equation from part (a) of problem 3-28 with tiles.

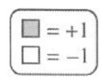
b. Read the Math Notes box at the end of this lesson to learn the "legal" moves you can make on an Equation Mat.

c. Solve the equation by making "legal" moves on your Equation Mat. Check your solution by evaluating the equation you wrote in part (a) of problem 3-28.

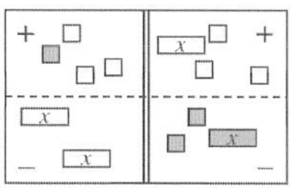
d. Build the equation from part (b) of problem 3-28. Solve the equation by using "legal" tile moves and check your solution.

3-30. Using algebra tiles on an Equation Mat, create a physical representation of the equation $-x+(-3)-(1+(-x))=-x+2-(2x+(-x))$. Use "legal" moves to solve the equation and check your answer.

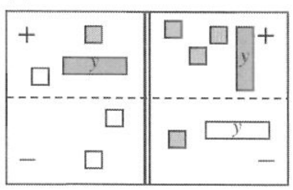
3-31. Write an equation (without simplifying) for each representation below. Build each equation on an Equation Mat, solve for the variable by making "legal" moves, and check your solution.



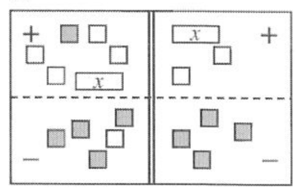
a.



b.



c.



3-32. Build each equation below. Then use "legal" moves to simplify it, solve for x or y, and check your solution. Write down the algebraic result of each step and the legal tile move you made to get there.

a. $-2x+2=-8$

b. $4x-2+x=2x+8+3x$

c. $3y-9+y=6$

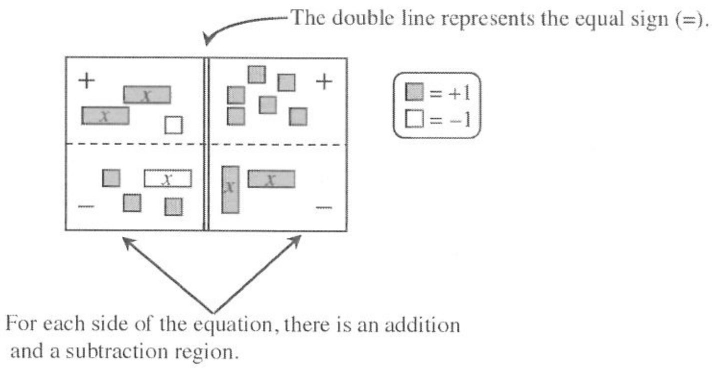
d. $9-(2+(-3y))=6+2y-(5+y)$

MATH NOTES

METHODS AND MEANINGS

Using Algebra Tiles to Solve Equations

Algebra tiles are a physical and visual representation of an equation. For example, the equation $2x+(-1)-(-x)-3=6-2x$ can be represented by the Equation Mat below.



An Equation Mat can be used to represent the process of solving an equation. The "legal" moves on an Equation Mat correspond with the mathematical properties used to algebraically solve an equation.

"Legal" Tile Move	Corresponding Algebra
Group tiles that are alike together.	Combine like terms.
Flip all tiles from subtraction region to addition region.	Change subtraction to "adding the opposite."
Flip everything on both sides.	Multiply (or divide) both sides by -1.
Remove zero pairs (pairs of tiles that are opposites) within a region of the mat.	A number plus its opposite equals zero.
Place or remove the same tiles on or from both sides.	Add or subtract the same value from both sides.
Arrange tiles into equal-sized groups.	Divide both sides by the same value.

3-33. Copy and simplify the following expressions by combining like terms. Using or drawing sketches of algebra tiles may be helpful.

a. $2x+3x+3+4x^2+10+x$

b. $4x+4y^2+y^2+9+10+x+3x$

c. $2x^2+30+3x^2+4x^2+14+x$

d. $20+5xy+4y^2+10+y^2+xy$

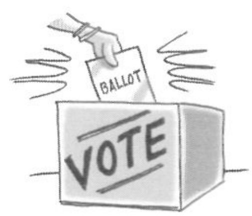
3-34. Solve each equation. Show the check to prove your answer is correct.

a. $3x+5-x=x-3$

b. $5x-(x+1)=5-2x$

3-35. Fisher thinks that any two lines must have a point of intersection. Is he correct? If so, explain how you know. If not, produce a **counterexample**. That is, find two lines that do not have a point of intersection and explain how you know.

3-36. Write and solve an equation for the following problem.



In the last election, candidate A received twice as many votes as candidate B. Candidate C received 15,000 fewer votes than candidate B. If a total of 109,000 votes were cast, how many votes did candidate A receive?

3-37. Evaluate the following expressions.

a. $10\frac{7}{9}+\left(-9\frac{2}{3}\right)$

b. $-10\frac{7}{10}-2\frac{3}{5}$

c. $\left(4\frac{1}{2}\right)\left(-3\frac{3}{10}\right)$

d. $-8\frac{3}{5}\div 1\frac{1}{5}$

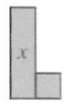
3-38. Find the equation of the line based on the table.

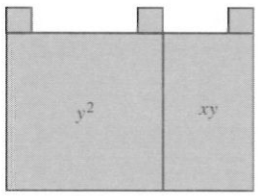
x	2	4	6	8
y	2	3	4	5

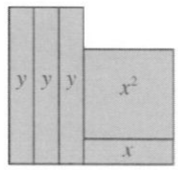
3-39. For each of the shapes formed by algebra tiles below:

- Sketch and label the shape on your paper and write an expression that represents the perimeter.
- Simplify your perimeter expression as much as possible.

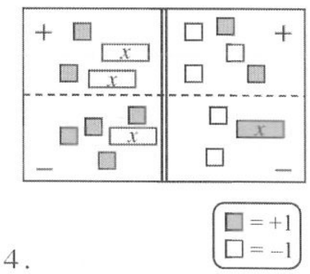
a. 

b. 

c. 

d. 

3-40. Translate the Equation Mat at right into an equation. Do not simplify your equation. Remember that the double line represents "equals."



3-41. Consider the rule $y=\frac{1}{2}x-4$.

a. Without graphing, find the x-intercept of $y=\frac{1}{2}x-4$.

b. Make a table and graph $y=\frac{1}{2}x-4$ on graph paper.

c. How could you find the x-intercept of $y=\frac{1}{2}x-4$ with your graph from part (b)? How would you find it with the table? Explain.

3-42. Evaluate each expression below for a when $a=\frac{2}{3}$, if possible.

a. $24a$ b. $3a$ c. $\frac{a}{0}$ d. $\frac{0}{a}$

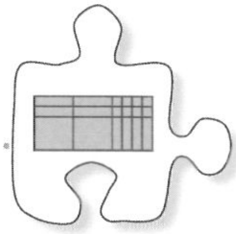
3-43. **Multiple Choice:** What is the slope of the line that goes through the points (–7, 10) and (1, 4)?

a. $\frac{3}{4}$ b. $-\frac{3}{4}$ c. 1 d. –1

3-44. Simplify each expression below, if possible.

a. $5x(3x)$ b. $5x+3x$ c. $6x(x)$ d. $6x+x$

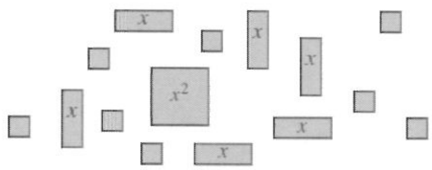
3.2.2 What can I do with rectangles?



Exploring an Area Model

In the last lesson, you used tiles to represent algebraic equations. Today you will use algebra tiles again, but this time to represent expressions using multiplication.

3-45. Your teacher will put this group of tiles on the overhead:



a. Using your own tiles, arrange the same group of tiles into one large rectangle, with the x^2 tile in the lower left corner. On your paper, sketch what your rectangle looks like.

b. What are the dimensions (length and width) of the rectangle you made? Label your sketch with its dimensions, then write the area of the rectangle as a *product*, that is, length · width.

c. The area of a rectangle can also be written as the sum of the areas of all its parts. Write the area of the rectangle as the *sum* of its parts. Simplify your expression for the sum of the rectangle's parts.

d. Write an equation that shows that the area written as a product is equivalent to the area written as a sum.

3-46. Your teacher will assign several of the expressions below. For each expression, build a rectangle using all of the tiles, if possible. Sketch each rectangle, find its dimensions, and write an expression showing the equivalence of the area as a *sum* (like x^2+5x+6) and as a *product* (like $(x+3)(x+2)$). If it is not possible to build a rectangle, explain why not.

a. x^2+3x+2

b. $6x+15$

c. $2x^2+7x+6$

d. $xy+x+y+1$

e. $2x^2+10x+12$

f. $2y^2+6y$

g. $y^2+xy+2x+2y$

h. $3x^2+4x+1$

i. $x^2+2xy+y^2+3x+3y+2$

j. $2xy+4y+x+2$

3-47. LEARNING LOG

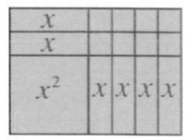
Make a rectangle from any number of tiles. Your rectangle must contain at least one of each of the following tiles: x^2, y^2, xy, x, y, and 1. Sketch your rectangle in your Learning Log and write its area as a product and as a sum. Explain how you know that the product and sum are equivalent. Title this entry "Area as a Product and as a Sum" and label it with today's date.

MATH NOTES

METHODS AND MEANINGS

Multiplying Algebraic Expressions with Tiles

The area of a rectangle can be written two different ways. It can be written as a *product* of its width and length or as a *sum* of its parts. For example, the area of the shaded rectangle at right can be written two ways:

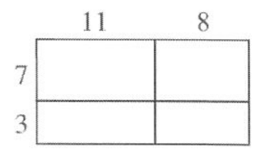


$$\underbrace{(x+4)}_{length}\underbrace{(x+2)}_{width} = \underbrace{x^2+6x+8}_{area}$$

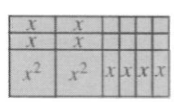
area as a product = area as a sum

Review & Preview

3-48. For the entire rectangle at right, find the area of each part and then find the area of the whole.



3-49. Write the area of the rectangle at right as a *product* and as a *sum*.



3-50. When solving $\frac{x}{6}=\frac{5}{2}$ for x, Nathan noticed that x is divided by 6.

a. What can he do to both sides of the equation to get x alone?

b. Solve for x. Then check your solution in the original equation.

c. Use the same process to solve this equation for x: $\frac{x}{10}=\frac{2}{5}$.

3-51. Jamila wants to play a game called "Guess My Line." She gives you the following hints: "Two points on my line are (1, 1) and (2, 4)."

a. What is the slope of her line? A graph of the line may help.

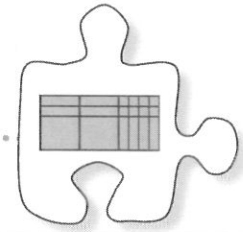
b. What is the y-intercept of her line?

c. What is the equation of her line?

3-52. A calculator manufacturer offers two different models for students. The company has sold 10,000 scientific calculators so far and continues to sell 1500 per month. It has also sold 18,000 graphical models and continues to sell 1300 of this model each month. When will the sales of scientific calculators equal the sales of graphical calculators?

3-53. On graph paper, make an $x \rightarrow y$ table and graph $y=2x^2-x-3$. Find its x- and y-intercepts.

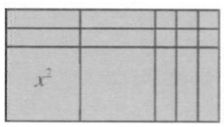
3.2.3 How can I rewrite a product?

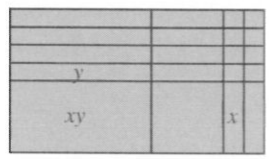


Multiplying Binomials and the Distributive Property

In Lesson 3.2.2, you made rectangles with algebra tiles and found the dimensions of the rectangles. Starting with the area of a rectangle as a sum, you wrote the area as a product. Today you will reverse the process, starting with the product and finding its area as a sum.

3-54. For each of the following rectangles, find the dimensions (length and width) and write the area as the *product* of the dimensions and as the *sum* of the tiles. Remember to combine like terms whenever possible.

a. 

b. 

3-55. Your teacher will assign your team some of the expressions below. Use your algebra tiles to build rectangles with the given dimensions. Sketch each rectangle on your paper, label its dimensions, and write an equivalence statement for its area as a product and as a sum. Be prepared to share your solutions with the class.

a. $(x+3)(2x+1)$

b. $2x(x+5)$

c. $x(2x+y)$

d. $(2x+5)(x+y+2)$

e. $(2x+1)(2x+1)$

f. $(2x)(4x)$

g. $2(3x+5)$

h. $y(2x+y+3)$

3-56. With your team, examine the solutions you found for parts (c), (e), (g), and (h) of problem 3-55. This pattern is called the Distributive Property. Multiply the following expressions without using your tiles and simplify. Be ready to share your process with the class.

a. $2x(6x+5)$

b. $6(4x+1)$

c. $3y(4x+3)$

d. $7y(10x+11y)$

3-57. CLOSED SETS

Whole numbers (positive integers and zero) are said to be a **closed set** under addition: if you add two whole numbers, you always get a whole number. Whole numbers are not a closed set under subtraction: if you subtract two whole numbers, you do not always get a whole number: $2 - 5 = -3$ (-3 is not a whole number).

a. Investigate with your team whether the integers are a closed set under addition, and whether the integers are a closed set under subtraction. Give examples. If you find that integers are closed under either of the operations, can you explain how you know they are closed for *all* integers?

b. Read the Math Notes box that follows. Are polynomials a closed set under addition? Are polynomials a closed set under subtraction? That is, if you add or subtract two polynomials, will you always get a polynomial as your answer? Give examples and explain how you know your answer is always true.

METHODS AND MEANINGS

MATH NOTES

Vocabulary for Expressions

A mathematical **expression** is a combination of numbers, variables, and operation symbols. Addition and subtraction separate expressions into parts called **terms**. For example, $4x^2 - 3x + 6$ is an expression. It has three terms: $4x^2$, $3x$, and 6. The **coefficients** are 4 and 3. 6 is called a **constant term**.

A one-variable **polynomial** is an expression which only has terms of the form:

$$(\text{any real number})x^{(\text{whole number})}$$

For example, $4x^2 - 3x^1 + 6x^0$ is a polynomial, so the simplified form, $4x^2 - 3x + 6$ is a polynomial.

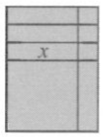
The function $f(x) = 7x^5 + 2.5x^3 - \frac{1}{2}x + 7$ is a polynomial function.

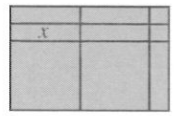
The following are not polynomials: $2^x - 3$, $\frac{1}{x^2-2}$, and $\sqrt{x-2}$.

A **binomial** is a polynomial with only two terms, for example, $x^3 - 0.5x$ and $2x + 5$.

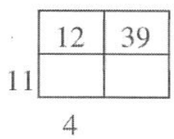
Review & Preview

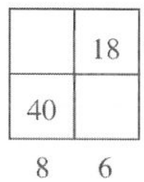
3-58. Examine the rectangles formed with tiles below. For each figure, write its area as a product of the width and length and as a sum of its parts.

a. 

b. 

3-59. Find the total area of each rectangle below. Each number inside the rectangle represents the area of that smaller rectangle, while each number along the side represents the length of that portion of the side.

a. 

b. 

3-60. Solve each equation below for x. Then check your solutions.

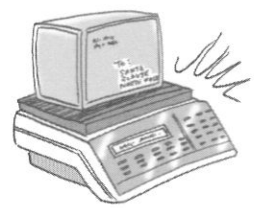
a. $\frac{x}{8}=\frac{3}{4}$

b. $\frac{2}{5}=\frac{x}{40}$

c. $\frac{1}{8}=\frac{x}{12}$

d. $\frac{x}{10}=\frac{12}{15}$

3-61. Mailboxes Plus sends packages overnight for \$5 plus \$0.25 per ounce. United Packages charges \$2 plus \$0.35 per ounce. Mr. Molinari noticed that his package would cost the same to mail using either service. How much does his package weigh?

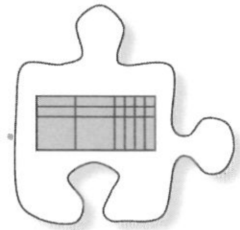


3-62. What is the equation of the line that has a y-intercept of $(0,-3)$ and passes through the point $(-9,-9)$?

3-63. Evaluate each expression.

a. $-7\frac{5}{6}+\left(-7\frac{1}{4}\right)$ b. $-8\frac{1}{2}-\left(-3\frac{1}{4}\right)$ c. $\left(-2\frac{3}{7}\right)(-7)$ d. $-2\frac{1}{8}\div\frac{1}{5}$

3.2.4 How can I generalize the process?



Using Generic Rectangles to Multiply

You have been using algebra tiles and the concept of area to multiply polynomial expressions. Today you will be introduced to a tool that will help you find the product of the dimensions of a rectangle. This will allow you to multiply expressions without tiles.

3-64. Use the Distributive Property to find each product below.

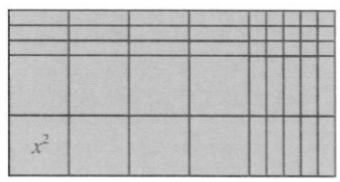
a. $6(-3x+2)$

b. $x^2(4x-2y)$

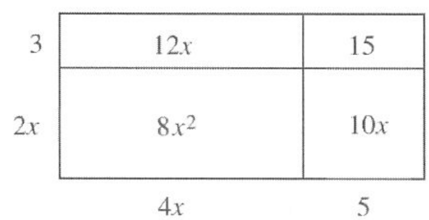
c. $5t(10-3t)$

d. $-4w(8-6k^2+y)$

3-65. Write the area as a *product* and as a *sum* for the rectangle shown at right.



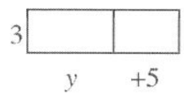
3-66. Now examine the following diagram. How is it similar to the set of tiles in problem 3-65? How is it different? Talk with your teammates and write down all of your observations.



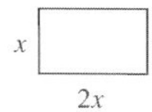
3-67. Diagrams like the one in problem 3-66 are referred to as **generic rectangles**. Generic rectangles allow you to use an area model to multiply expressions without using the algebra tiles. Using this model, you can multiply with values that are difficult to represent with tiles.

Draw each of the following generic rectangles on your paper. Then find the area of each part and write the area of the whole rectangle as a *product* and as a *sum*.

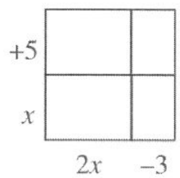
a.



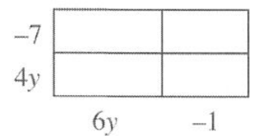
b.



c.



d.



e. How did you find the area of the individual parts of each generic rectangle?

3-68. Multiply and simplify the following expressions using either a generic rectangle or the Distributive Property. For part (a), verify that your solution is correct by building a rectangle with algebra tiles.

a. $(x+5)(3x+2)$

b. $(2y-5)(5y+7)$

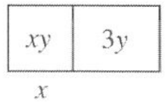
c. $3x(6x^2-11y)$

d. $(5w-2p)(3w+p-4)$

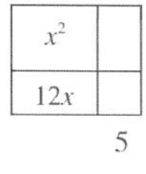
3-69. THE GENERIC RECTANGLE CHALLENGE

Copy each of the generic rectangles below and fill in the missing dimensions and areas. Then write the entire area as a product and as a sum. Be prepared to share your reasoning with the class.

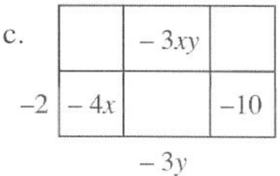
a.



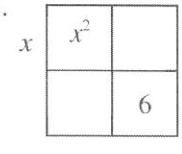
b.



c.



d.



METHODS AND MEANINGS

Properties of Real Numbers

MATH NOTES

The legal tiles moves have formal mathematical names, called the **properties of real numbers**.

The **Commutative Property** states that when *adding* or *multiplying* two or more number or terms, order is not important. That is:

$a+b=b+a$ For example, $2+7=7+2$

$a \cdot b=b \cdot a$ For example, $3 \cdot 5=5 \cdot 3$

However, *subtraction* and *division* are not commutative, as shown below.

$$7-2 \neq 2-7 \text{ since } 5 \neq -5$$

$$50 \div 10 \neq 10 \div 50 \text{ since } 5 \neq 0.2$$

The **Associative Property** states that when *adding* or *multiplying* three or more number or terms together, grouping is not important. That is:

$(a+b)+c=a+(b+c)$ For example, $(5+2)+6=5+(2+6)$

$(a \cdot b) \cdot c=a \cdot(b \cdot c)$ For example, $(5 \cdot 2) \cdot 6=5 \cdot(2 \cdot 6)$

However, *subtraction* and *division* are not associative, as shown below.

$(5-2)-3 \neq 5-(2-3)$ since $0 \neq 6$ $(20 \div 4) \div 2 \neq 20 \div(4 \div 2)$ since $2.5 \neq 10$

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

$a+0=a$ For example, $6+0=6$

The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

$1 \cdot a=a$ For example, $1 \cdot 6=6$

The **Additive Inverse Property** states that for every number a there is a number $-a$ such that $a+(-a)=0$. A common name used for the additive inverse is the **opposite**. That is, $-a$ is the opposite of a. For example, $3+(-3)=0$ and $-5+5=0$.

The **Multiplicative Inverse Property** states that for every nonzero number a there is a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a}=1$. A common name used for the multiplicative inverse is the **reciprocal**. That is, $\frac{1}{a}$ is the reciprocal of a. For example, $6 \cdot \frac{1}{6}=1$.

Review & Preview

3-70. Use a generic rectangle to multiply the following expressions. Write each solution both as a sum and as a product.

a. $(2x+5)(x+6)$ b. $(m-3)(3m+5)$

c. $(12x+1)(x^2-5)$ d. $(3-5y)(2+y)$

3-71. Find the rule for the pattern represented at right.

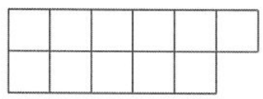
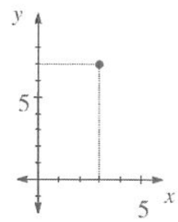
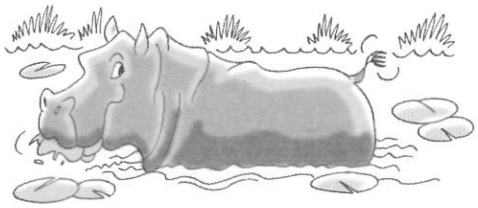


Figure 1



3-72. Harry the Hungry Hippo is munching on the lily pads in his pond. When he arrived at the pond, there were 20 lily pads, but he is eating 4 lily pads an hour. Heinrick the Hungrier Hippo found a better pond with 29 lily pads! He eats 7 lily pads every hour.



a. If Harry and Heinrick start eating at the same time, when will their ponds have the same number of lily pads remaining?

b. How many lily pads will be left in each pond at that time?

3-73. Graph each equation below on the same set of axes and label the point of intersection with its coordinates.

$$y=2x+3 \qquad y=x+1$$

3-74. Are the odd numbers a closed set under addition? Justify your conclusion.

3-75. Simplify each of the expressions below. Your final simplification should not contain negative exponents.

a. $(5x^3)(-3x^{-2})$ b. $(4p^2q)^3$ c. $\frac{3m^7}{m^{-1}}$